

Empirical study on the integration of total least squares and radial basis function neural network for coordinate transformation

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ABSTRACT

Total Least Squares (TLS) is noted to be a solution approach to solving several geodetic problems. The method has the ability to estimate unknown quantities that are useful for many geodetic applications. Hence, the main objective of this study was to improve the estimation performance of TLS via Radial Basis Function Neural Network (RBFNN) in coordinate transformation. This hybrid approach called TLS-RBFNN was applied to Ghana geodetic reference network, which has a coverage area of 79857 km² representing 33.5% of the total land mass (238540 km²). A comparative performance analysis of TLS, RBFNN and TLS-RBFNN was carried out using Root Mean Square Horizontal Error (RMSHE) and Standard Deviation (SD). Based on the testing results, it was found that the TLS-RBFNN improved the transformation accuracy of RBFNN and TLS by 20.2% and 37.3% based on the RMSHE. In addition, it was observed that the TLS-RBFNN improved the transformation precision based on SD by 0.37% and 8.52%, respectively. Furthermore, the Bayesian Information Criterion (BIC) applied confirmed the superiority of the hybrid approach than using TLS and RBFNN as independent transformation methods. Consequently, the hybrid approach is recommended for enhanced coordinate transformation results in Ghana geodetic reference network.

Keywords: Total Least Squares, Artificial Neural Network, Coordinate Transformation, Global Navigation Satellite System

INTRODUCTION

Global Navigation Satellite System (GNSS) has been a major source of technology used for Earth observations since its inception. GNSS, particularly Global Positioning System (GPS), has become increasingly popular over the years, due partly to the multitude of problems associated with the conventional surveying techniques. It is well acknowledged that GNSS acquired data is based on geocentric system of reference and thus cannot be directly used in astro-geodetic datums. Hence, due to change in datum position, size and shape, coordinate transformation between two geodetic

datums qualifies it to be a good choice. Thus, the transformation attempts to provide a unifying approach that bridges the gap between the different datums.

Several conventional transformation methods such as Bursa-Wolf (Bursa, 1962; Wolf, 1963), Molodensky-Badekas (Molodensky *et al.*, 1962; Badekas, 1969), geocentric translation model, standard Molodensky, Abridged Molodensky, polynomial and multiple regression (Applebaum, 1982; Featherstone, 1997; Fraser and Yamakawa, 2004; Newsome and Harvey, 2003) have been widely applied in literature. These mentioned techniques have peculiar characteristics they have been employing to accomplish the coordinate transformation tasks. Thus, by utilising set of transformation parameters.

Over the years, numerous techniques have been proposed and used to determine the transformation parameters. In practice, the most common alternatives may be categorised into least squares algorithms, partitioning methods, Ill-posed approach, quaternions approach and Procrustes algorithm (Collier *et al.*, 1998; Soler and Snay, 2003; Grafarend and Awange, 2003; Shen *et al.*, 2006; Felus and Schaffrin, 2005; Lippus, 2004). However, the popularity of Artificial Neural Network (ANN) methodology has been growing in a wide variety of areas in geodesy and geomatic engineering. Its efficacy as a coordinate transformation technique is well documented. Literature confirms that the ANN approaches could produce reasonable and promising results that are more satisfactory than the empirical affine, conformal and projective transformation methods (Tierra *et al.*, 2009; Kumi-Boateng and Ziggah, 2017; Ziggah *et al.*, 2016; Ziggah *et al.*, 2018; Ziggah, & Laari, 2018; Ziggah *et al.*, 2019a; Ziggah *et al.*, 2019b; Gullu and Narin, 2019). The strength of the ANN has been credited to its ability to effectively reduce the distortion and heterogeneity in spatial data related to the different geodetic datums. Moreover, the self-adaptive feature of the ANN enables it to appropriately identify hidden patterns in a data set to build the prediction model.

In the last few years, combining ANN and empirical methods to create a hybrid approach is becoming very popular among geoscience researchers. Examples of some application areas include landslide studies (Huang *et al.*, 2016), hydrological studies (Tiwari *et al.*, 2016), meteorology (Ghorbani *et al.*, 2016) to mention but a few. Literature reveals that the hybrid approach offers better computational efficiency than separately applying the methods. Whereas the hybrid concept is becoming much appreciated, none of the existing studies in coordinate transformation fully address the issue of combining empirical transformation procedure and ANN technique or any other soft computing techniques. The dominance of ANN to solve several geodetic problems (e.g. coordinate transformation, tide modelling, gravity field modelling, orbit determination, digital terrain height estimation, crustal deformation, GNSS error modelling, etc.) makes it important computational tool (Tierra and De Freitas, 2005; Kaftan *et al.*, 2011; Liao *et al.*, 2012; Salim *et al.*, 2015; Lei *et al.*, 2015; Okwuashi and Ndehedehe, 2015; Huang *et al.*, 2016; Razin and Voosoghi, 2017; Gullu and Narin, 2019).

Therefore, motivated by the successful application of ANN, the main focus of this study is to explore the potential of integrating Total Least Squares (TLS) and Radial Basis Function Neural Network (RBFNN) in coordinate transformation process. The choice of these methods was based on its frequent use, simplicity in application and computational efficiency. The TLS-RBFNN integrated approach was tested in Ghana geodetic reference network to perform coordinate transformation between the global WGS84 and Ghana's War Office 1926 ellipsoid. Such integration approach is significant for Ghana because of the astro-geodetic datum used for

surveying and mapping works. In addition, with all the attendant problems of the astro-geodetic datums in mind (Varga, Grgić, & Bašić, 2017; Poku-Gyamfi, 2009), the need to explore and test the potential of different coordinate transformation procedures in the Ghana geodetic reference network is reasonable. In continuance of that, the TLS and RBFNN when combined will both use their strengths and weaknesses to complement each other. Thus, the hybrid approach will combine their function approximation and nonlinear modelling capabilities.

Moreover, using TLS can improve the performance of the RBFNN in many aspects such as training speed, reduction in the size of the network leading to fast convergence and satisfactory transformation results. The proposed hybrid approach could be categorised as a combination of knowledge-based system and empirical model. The idea behind this study is not to take away the significance of the empirical transformation models, but to demonstrate from a practical perspective how combining those with ANN can be more effective in solving coordinate transformation problems. Evidently, this study shows the potential and the resulting consequences of using such hybrid approach as a practical tool for coordinate transformation in Ghana geodetic reference network.

MATERIALS AND METHODS

Study Area and Data Used

Ghana is a West African country bounded by latitudes $4^{\circ}30'$ N and 11° N, and longitudes 3° W and 1° E (Mugnier, 2000). The country uses the Accra 1929 datum for its geospatial activities. The reference ellipsoid of the Accra 1929 datum is the War Office 1926. This reference ellipsoid has its origin at latitude $5^{\circ} 23' 43.33''$ N and longitude $0^{\circ} 11' 52.3''$ W with semi-major axis $a = 6378299.99899832$ m, semi minor axis $b = 6356751.68824042$ m and inverse flattening $f = 296$ (Poku-Gyamfi, 2009; Ayer, 2008; Ayer and Fosu, 2008). For the purposes of land surveys, the Transverse Mercator projection has been adopted to derive projected grid coordinates in Easting and Northing. The Transverse Mercator has its origin at longitude $01^{\circ} 00'$ W and latitude $04^{\circ} 40'$ N with false Easting value of 274319.736 m added to all Y coordinates to avoid negative coordinates and setting the false Northing to zero (Mugnier, 2000; Poku-Gyamfi, 2009). A used scale factor of 0.99975 was found to eliminate the scale distortion effect within the country but this can exceed the projection values only at the borders (extreme ends) of the country. Since Accra 1929 datum is non-geocentric but GPS acquired data based on WGS84 is geocentric, localisation of the GPS data will require coordinate transformation. In this study, two sets of 19 co-located points were collected based on the War Office 1926 $(\varphi, \lambda, h)_{WAR}$ and WGS84 $(\varphi, \lambda, h)_{WGS84}$; where φ is the geodetic latitude, λ is the geodetic longitude and h is the ellipsoid height. The point distribution of the data used for the analysis is shown in Figure 1. These data sets were provided by the Ghana Survey and Mapping Division of Lands Commission from the Land Administration Project. It is important to note that these co-located points are the only available data set and those for the northern part and the rest of the country are yet to be observed.

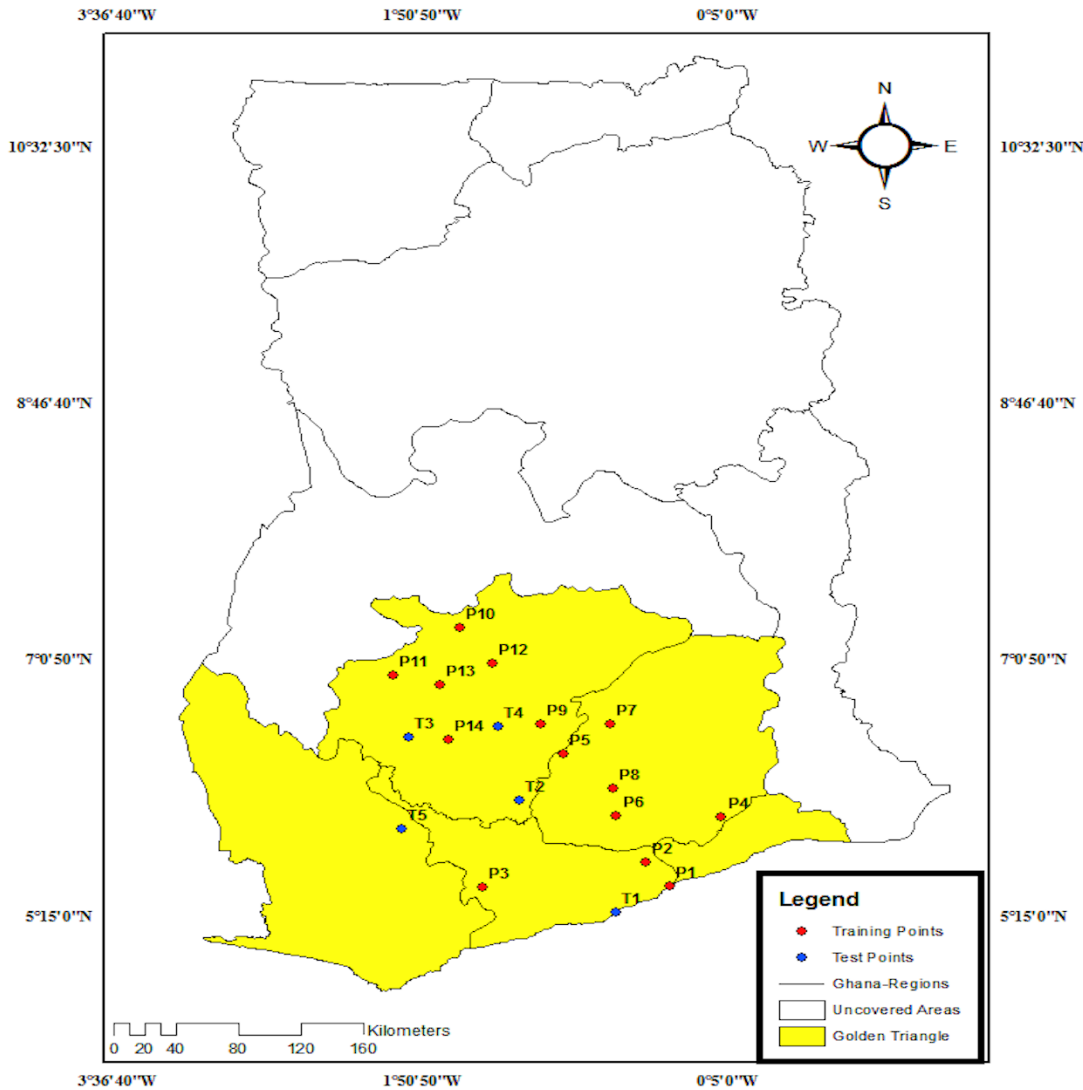


FIGURE 1. Co-Located Points Distribution in the Study Area

APPLIED METHODS

Total Least Squares

The generic least squares type is the Ordinary Least Squares (OLS) with the implicit assumption that the errors in the design matrix are zero but the only errors present can be found in the observation vector. However, in geodetic practice, all observed data (coordinates) may suffer some form of errors. In order to correct for such errors, a more pragmatic TLS approach is needed (Felus and Schaffrin, 2005; Akyilmaz, 2007; Okwuashi and Eyoh, 2012). Consequently, this study applied the TLS approach for the coordinate transformation due to its superiority to the OLS in

producing precise coordinate transformation results as indicated by several authors (Akyilmaz, 2007; Felus and Schaffrin, 2005; Okwuashi and Eyoh, 2012).

Golub and Van Loan (1980) introduced the TLS approach as a solution technique to estimate an over-determined system of linear equations expressed in Equation (1) as

$$BX = K \quad (1)$$

where $B \in R^{m \times n}$ and $K \in R^{m \times d}$ are the given datasets, $X \in R^{n \times d}$ is the unknown parameters to be determined, m is the number of observations, n is the number of unknowns and $m \geq n$. To estimate X in Equation (1), the design matrix (B) is considered to be affected by random errors expressed in Equation (2) (Akyilmaz, 2007) as

$$(B + v_B)X = K + v_K, \text{ rank}(B) = m < n \quad (2)$$

It must be known that both v_K (error vector of observations) and v_B (error matrix of data matrix) in Equation (2) are presumed to have independent and equivalent distributed rows with zero mean and equal variance (Akyilmaz, 2007). To minimize the errors in the data, the TLS approximation (Equation 3) is iteratively applied.

$$\min \left\| [B; K] - [\hat{B}; \hat{K}] \right\|_F, [\hat{B}; \hat{K}] \in R^{m \times (n+d)}, \text{ subject to: } \hat{K} \in R(\hat{B}) \quad (3)$$

where m and n are the same as defined in Equation (1); \hat{B} is the new estimated data matrix; \hat{K} is the new estimated observation vector; and $\| \cdot \|_F$ is the Frobenius norm of $m \times n$ matrix. Equation (3) iteration continues until a minimizing matrix $[\hat{B}; \hat{K}]$ is found such that any \hat{X} (estimated unknown parameters) satisfying $\hat{B}\hat{X} = \hat{K}$ is the TLS solution and $[\Delta\hat{B}; \Delta\hat{K}] = [B; K] - [\hat{B}; \hat{K}]$ is the corresponding TLS correction (Akyilmaz, 2007; Golub and Van Loan, 1980; Okwuashi and Eyoh, 2012). It is worth mentioning that the TLS solution is usually determined through the functional relation expressed in Equation (4) as

$$[B; K] \begin{bmatrix} \hat{X}^T \\ -1 \end{bmatrix}^T \approx 0. \quad (4)$$

In this study, the Singular Value Decomposition (SVD) approach was applied on the matrix $[B; K]$ to solve the TLS problem. Using the SVD enables the matrix $[B; K]$ to be investigated to know whether it is rank deficient or not. The SVD mathematical representation of the matrix $[B; K]$ (Van Huffel and Vandewalle, 1991) is given in Equation (5) as

$$[B; K] = USV^T \quad (5)$$

where $U = [U_1; U_2]$, $U_1 = [u_1, \dots, u_n]$, $U_2 = [u_{n+1}, \dots, u_m]$, $u_i \in R^m$, $U^T U = I_m$,

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}_d^n = [v_1, \dots, v_{n+d}] v_i \in R^{n+d}, V^T V = I_{n+d},$$

$$S = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} = \text{diag}(\sigma_1, \dots, \sigma_{n+1}) \in R^{m \times (n+d)}, t = \min\{m - n, d\},$$

$$\Sigma_1 = \text{diag}(\sigma_1, \dots, \sigma_n) \in R^{n \times n},$$

$$\Sigma_2 = \text{diag}(\sigma_{n+1}, \dots, \sigma_{n+t}) \in R^{(m-n) \times d}, \text{ and } \sigma_1 \geq \dots \geq \sigma_{n+1} \geq 0.$$

The σ_i are the singular values of B and $[B:K]$, and the vectors u_i and v_i are the i th left and right singular vector of B and $[B:K]$, respectively. A TLS solution exists if and only if V_{22} is non-singular and the solution is unique if and only if $\sigma_n \neq \sigma_{n+1}$. The TLS solution is therefore expressed as

$$\hat{X}_{tls} = -V_{12} V_{22}^{-1} \text{ and the corresponding TLS correction matrix is expressed in Eq. (6) as}$$

$$\Delta C_{TLS} = [\Delta B_{TLS}; \Delta K_{TLS}] = -U \text{diag}(0, \Sigma_2) V^T. \quad (6)$$

Radial Basis Function Neural Network

Broomhead and Lowe (1988) proposed RBFNN technique which processes information by using norms and radial basis functions. This network creates an alternative means of applying arbitrary input-output patterns for function approximation problems (Deyfrus, 2005). It has a feedforward topology consisting of three layers; the input, hidden and output that are completely linked together. Figure 2 shows the RBFNN architecture with inputs (X_1, X_2, \dots, X_d), radial basis function ($\varphi_1, \varphi_2, \dots, \varphi_N$), weight (W_1, W_2, \dots, W_N) and output (y) respectively.

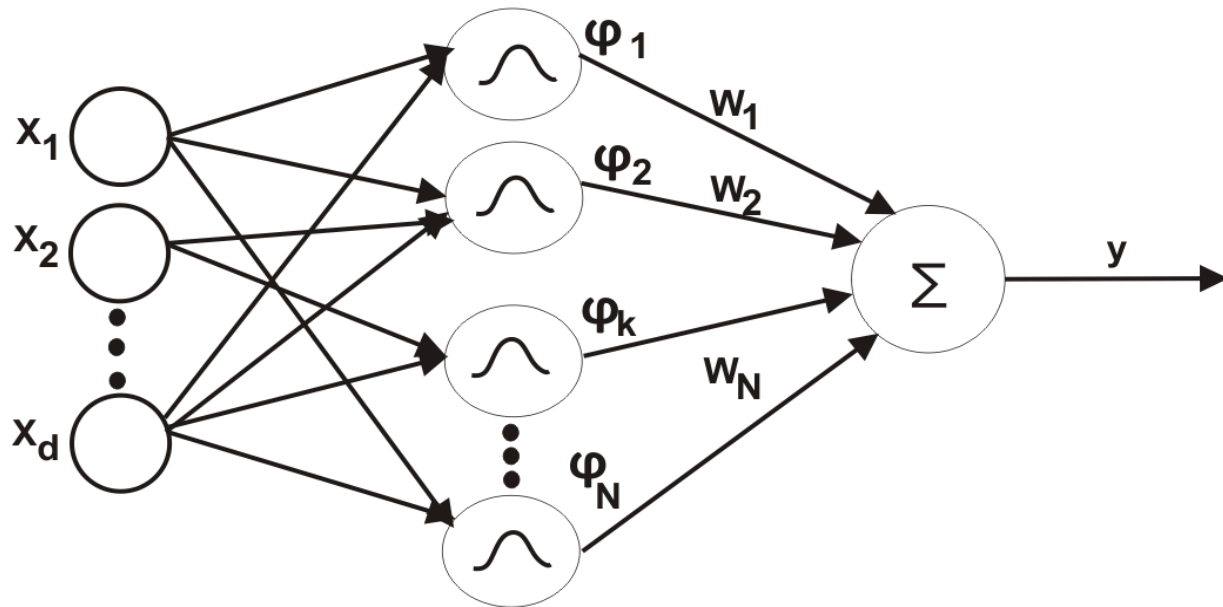


FIGURE 2. Radial Basis Function Neural Network Structure

This study adopted the supervised learning algorithm to train the RBFNN. A code written in MATLAB environment was used to carry out the RBFNN training. In the RBFNN scheme, the input layer comprises the data that are submitted into the network by unweighted connections. These input nodes are then transmitted into the hidden layer chamber by a non-linear activation function. Within the hidden layer, each neuron computes a Euclidean norm that represents the distance between the input to the network and the position of the neuron called the centre. This is then inserted into a radial basis transfer function which estimates and outputs the activation of the

neuron (Deyfrus, 2005). This study applied the Gaussian activation function expressed in Equation (7) as

$$\beta_i(X) = \exp\left[-\frac{\|X - \mu_j\|^2}{2\sigma_i^2}\right] \quad (7)$$

where $\beta_i(X)$ denote the hidden layer output of the i th unit, X is the input vector, μ_j is the centre of the Gaussian function for the hidden node j , σ_i is a spread parameter for regulating smoothness properties of the Gaussian function and $\|X - \mu_j\|$ is the Euclidean norm. When calculations are completed, the output layer containing the identity activation function then uses the weighted sum of the radial basis function layer as propagation function.

The estimated output layer results from the RBFNN could be represented by Equation (8) (Jain, Singh, & Srivastava, 2011) as

$$y = \sum_{i=1}^M w_i \beta_i(X) + w_o, \quad (8)$$

where M is the number of radial basis function and w represent the weight of the network. The mean squared error (MSE) for the X th data was estimated using Equation (9) (Jain *et al.*, 2011).

$$MSE = \frac{1}{2} (y_o - y_p)^2 \quad (9)$$

where y_o and y_p are the actual output and target for the X th data, respectively. The training process was then repeated for all training data. Equation (10) (Jain *et al.*, 2011) was applied to estimate the error function (E_k).

$$E_k = \sum_{X=1}^{X^{total}} MSE = \frac{1}{2} \sum_{X=1}^{X^{total}} (y_o - y_t)^2 \quad (10)$$

where X^{total} represents the total number of training data. The connection weight, w_{pi} which minimizes E_k was updated using Equation (11).

$$w_{pi}(K+1) = w_{pi}(K) + \Delta w_{pi}(K) \quad (11)$$

where $\Delta w_{pi}(K)$ is given by Equation (12) as

$$\Delta w_{pi}(K) = \eta(K) \sum_{X=1}^{X^{total}} \delta_X A_{Xi} + \alpha \Delta w_{pi}(K-1). \quad (12)$$

where $\eta(K)$ is the learning rate at K th iteration, δ_X is the error signal of the X th data, α is the momentum parameter and A_{Xi} denote the output vector of the hidden layer neuron i for all the input data. This RBFNN training process was repeated till the network error reaches an acceptable value.

Proposed Hybrid Model

Local geodetic networks are highly distorted compared to the geocentric reference systems. The distortion is partly contributed by the conventional surveying techniques such as triangulation, traversing, resection and astronomical observations used in establishing the local geodetic network. Thus introducing distortions, systematic or observational errors within the network. Furthermore, the adjustment procedures used to adjust the local geodetic networks were of lower accuracies (Varga *et al.*, 2017 and references therein). This supports the point made in Featherstone

(1997) and Grgić *et al.*, (2015) that the present conformal transformation models often times are unable to accurately reduce the distortion effects on the final transformation outputs.

After careful review of existing research works pertaining to Ghana geodetic network, the author realised that in many applications of the conformal transformation models, large residuals are often produced. Hence, in this study an attempt has been made to explore the integration of TLS and RBFNN in the transformation process. The motive is to determine the viability of such integrated approach for coordinate transformation in Ghana. Figure 3 denotes a summary of the proposed hybrid approach.

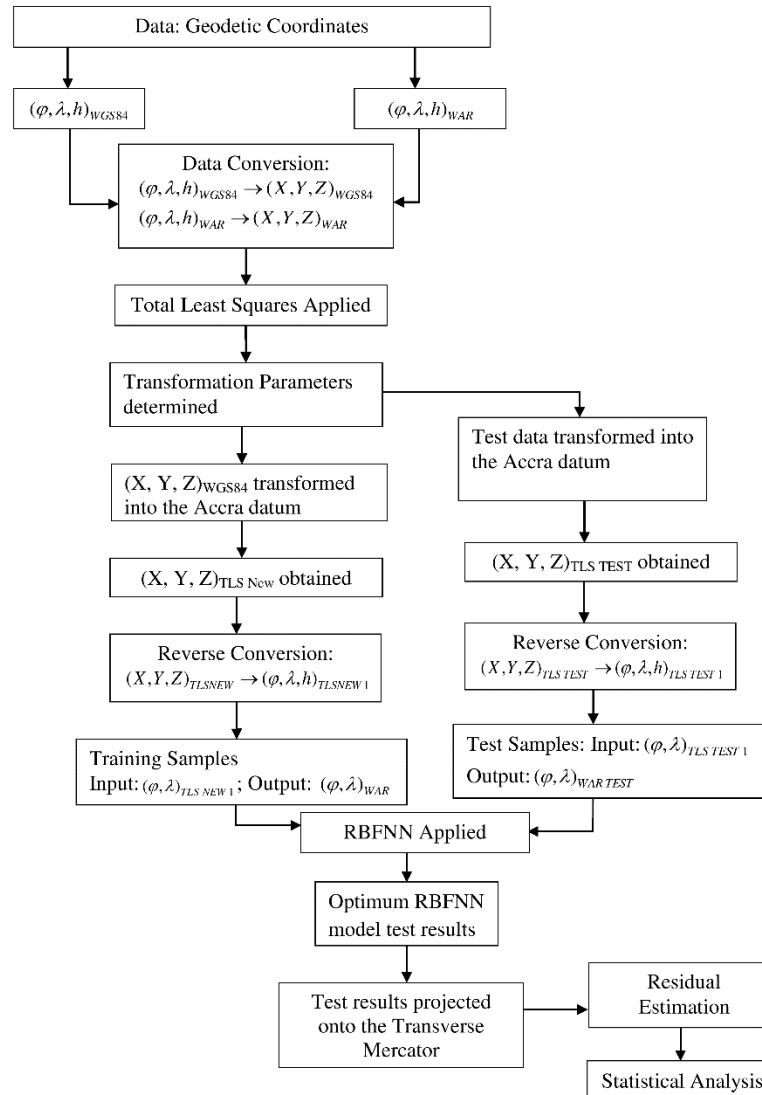


FIGURE 3. Flow Chart of the Proposed Hybrid Approach

Specific detail on how the hybrid approach was developed and implemented is presented as follows:

Step 1: Converting Geodetic Coordinates to Cartesian Coordinates

The geodetic coordinates of co-located points in both WGS84 $(\varphi, \lambda, h)_{WGS84}$ and War Office 1926 $(\varphi, \lambda, h)_{WAR}$ were converted into Cartesian coordinates using Equation (13) (Hofmann-Wellenhof and Moritz, 2006). The results from Equation (13) for the WGS84 and War Office 1926 are designated in this study as $(X, Y, Z)_{WGS84}$ and $(X, Y, Z)_{WAR}$, respectively.

$$\begin{aligned} X &= (S + h) \cos \varphi \cos \lambda \\ Y &= (S + h) \cos \varphi \sin \lambda \\ Z &= [S(1 - e^2) + h] \sin \varphi \end{aligned} \quad (13)$$

where S is the radius of curvature in the prime vertical as defined in Equation (14).

$$S = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}} \quad (14)$$

a and e are the semi-major axis and first eccentricity of the reference ellipsoid.

Step 2: Determine Coordinate Transformation Parameters

Out of the 19 co-located points, 14 $(X, Y, Z)_{WGS84}$ and $(X, Y, Z)_{WAR}$ were purposefully chosen as reference points set defined as $P = (P_1, P_2, \dots, P_{14})$. These points were applied to determine the transformation parameters for transforming WGS84 coordinates into Ghana War Office 1926. The remaining five points served as the independent reference test set defined as $T = (T_1, T_2, \dots, T_5)$ for the model validation (see Figure 1). The seven parameter Bursa-Wolf (Bursa, 1962; Wolf, 1963) transformation model (Equation 15) was applied. It is known that for 14 common points, 35 observation equations more than the seven unknown parameters to be estimated. This clearly creates an over-determined system of linear equations. The TLS technique can be applied to such situations to achieve the best estimates of the unknown parameters. Hence, the TLS procedure based on the Singular Value Decomposition (SVD) approach (Equation 5) was applied to Equation (15) to determine the unknown transformation parameters. These determined parameters consisted of three translation vectors, three rotational parameters and one scale factor.

$$\begin{bmatrix} X_{WAR} \\ Y_{WAR} \\ Z_{WAR} \end{bmatrix} = \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} + \eta R \begin{bmatrix} X_{WGS84} \\ Y_{WGS84} \\ Z_{WGS84} \end{bmatrix} \quad (15)$$

where T_X , T_Y and T_Z are the translation vectors along X-, Y- and Z-axes respectively of the two reference systems. η is the scale factor and R is the total rotational matrix (product of the rotation angles).

Step 3: Apply the Determined Transformation Parameters

The seven transformation parameters determined in Step 2 were then used to transform the 14 $(X, Y, Z)_{WGS84}$ into the War Office 1926 datum. These newly transformed Cartesian coordinates are denoted in this study as $(X, Y, Z)_{TLS\ NEW}$. Similarly, the transformation parameters were also applied to transform the five independent test points into War Office 1926 datum. These transformed test coordinates are represented in this work as $(X, Y, Z)_{TLS\ TEST}$.

Step 4: Convert Cartesian Coordinates to Geodetic Coordinates

The $(X, Y, Z)_{TLS\ NEW}$ and $(X, Y, Z)_{TLS\ TEST}$ obtained in Step 3 were then converted to geodetic coordinates using Bowring inverse method (Equations 16-18) (Bowring, 1976). The converted coordinates were denoted in this study as $(\varphi, \lambda, h)_{TLS\ NEW_1}$ and $(\varphi, \lambda, h)_{TLS\ TEST_1}$, respectively.

$$\varphi = \tan^{-1} \left[\frac{Z + \varepsilon^2 b \sin^3 \xi}{p - e^2 a \cos^3 \xi} \right] \quad (16)$$

$$\lambda = \tan^{-1} \left[\frac{Y}{X} \right] \quad (17)$$

$$h = p \cos \varphi + Z \sin \varphi - a \sqrt{1 - e^2 \sin^2 \varphi} \quad (18)$$

where ξ is the parametric latitude (Equation (19)), b is the semi-minor axis of the ellipsoid, p is the perpendicular distance from the rotational axis (Equation (20)) and ε (Equation (21)) is the second eccentricity.

$$\xi = \tan^{-1} \left[\frac{aZ}{bP} \right] \quad (19)$$

$$p = \sqrt{X^2 + Y^2} \quad (20)$$

$$\varepsilon = \frac{e^2}{1 - e^2} \quad (21)$$

Step 5: Fuse TLS Results into RBFNN (Hybrid model)

The reverse conversion in Step 4 was necessary in order to select the input variables that produces the best results when the RBFNN was trained. This is important because it has been found that the appropriate input parameters used have influence on the prediction accuracy of the ANN model (Dreiseitl and Ohno-Machado 2002; Ismail *et al.*, 2012). In the light of that, several input parameters were tried and tested to determine the one that can produce the best transformation results. Firstly, $(X, Y, Z)_{\text{TLS NEW}}$ obtained in Step 3 was used as the input data and its corresponding $(X, Y, Z)_{\text{WAR}}$ was used as the output data in the RBFNN training. In the second scenario, $(\varphi, \lambda, h)_{\text{TLS NEW1}}$ obtained in Step 4 was used as the input data and $(\varphi, \lambda, h)_{\text{WAR}}$ as the output data. In the third scenario, The RBFNN was trained using $(\varphi, \lambda)_{\text{TLS NEW1}}$ obtained in Step 4 as the input variables and $(\varphi, \lambda)_{\text{WAR}}$ as the output variable. It was found that using the third case yielded better fusion of TLS and RBFNN results. Thus, the hybrid model was formed by using the TLS transformed coordinates as the input data with its corresponding coordinates in the Ghana War Office 1926 as the output data. When the training process was over, the hybrid calculation model for coordinate transformation in the study area was developed.

Step 6: Transforming Coordinates with Hybrid Model (TLS-RBFNN)

For any position in the study area, when its transformed geodetic coordinates are computed using the TLS algorithm (Steps 1 to 4), then its improved transformed coordinates can be calculated using the TLS-RBFNN developed in Step 5. In furtherance of this, the five testing points were used to test the capability of the TLS-RBFNN model developed. In that case, $(\varphi, \lambda)_{\text{TLS TEST1}}$ obtained in Step 4 was used as the input data in the optimum trained hybrid model and its corresponding known coordinates in the War Office 1926 denoted as $(\varphi, \lambda)_{\text{WAR TEST}}$ served as the target output. The new predicted output from the optimum trained hybrid model is represented as $(\varphi, \lambda)_{\text{WAR NEW}}$. The $(\varphi, \lambda)_{\text{WAR NEW}}$ geodetic coordinates were then projected onto the Transverse

Mercator 1° NW to obtain 2D projected grid coordinates in Easting (E) and Northing (N) using equations in Dzidefo (2011). The computed projected grid coordinates were then compared with the known grid coordinates for statistical analysis. The map projection conducted was important because Ghana uses the 2D projected grid coordinate system for its surveying and mapping related activities.

Step 7: Statistical Analysis

In order to check the efficiency of TLS, RBFNN and TLS-RBFNN, residuals between the known and computed projected grid coordinates were estimated. Statistical evaluation metric presented in Section 4 were then used to quantify these residuals generated for results interpretation.

Model Performance Criteria

Performance criteria indicators (PCIs) such as Horizontal Error (HE), Root Mean Square Horizontal Error (RMSHE), Mean Horizontal Error (MHE), Standard deviation (SD), Maximum Error (Max Error) and Minimum Error (Min Error) were used to check the adequacy of the methods applied. The mathematical representations (Ziggah *et al.*, 2019a) of the various PCIs are given by Equations (22) to (27).

$$HE = \sqrt{(E_{t_i} - E_{o_i})^2 + (N_{t_i} - N_{o_i})^2} = \sqrt{\Delta E^2 + \Delta N^2} \quad (22)$$

$$RMSHE = \sqrt{\frac{\sum_{i=1}^n (HE_i)^2}{n}} \quad (23)$$

$$MHE = \frac{1}{n} \sum_{i=1}^n HE_i \quad (24)$$

$$SD = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (HE - \overline{HE})^2} \quad (25)$$

$$\text{Max HE} = \max (HE_i)_{i=1}^n \quad (26)$$

$$\text{Min HE} = \min (HE_i)_{i=1}^n \quad (27)$$

where n is the total number of test data used, E_{t_i} and N_{t_i} represent the known Easting and Northing grid coordinates while E_{o_i} and N_{o_i} are the computed grid coordinates given by the TLS, RBFNN and TLS-RBFNN, respectively. \overline{HE} is the average value of the horizontal error.

RESULTS AND DISCUSSION

Model Performance Analysis

The RBFNN model which gave the optimum results was [2-14-2]. Thus, 2 inputs consisting of $(\varphi_{WGS84}, \lambda_{WGS84})$, 14 hidden neurons and $(\varphi_{WAR}, \lambda_{WAR})$ as the 2 outputs. The optimum TLS-RBFNN model comprises [2-14-2]. That is, $(\varphi, \lambda)_{TLS\ NEW\ 1}$ is the 2 inputs data, 14 hidden neurons in the hidden layer and $(\varphi_{WAR}, \lambda_{WAR})$ as the 2 outputs. In this study, the criterion to determine the optimum RBFNN and TLS-RBFNN model was based on the concept of ANN generalisation. It is a well-known fact that generalisation provides a more convincing estimate on the validity of ANN

models (Urolagin *et al.*, 2011). That is, the ability of ANN model to perform well when untrained data is presented to the network. To achieve this, the Mean Square Error (MSE) criterion was employed. That is, the model structure that produced the least MSE when the test data was fed into the RBFNN and TLS-RBFNN trained models was selected as the optimum model. For the TLS, the derived transformation parameters and their respective standard deviations of the 14 common points for transforming WGS84 coordinates to War Office 1926 datum are presented in Table 1.

TABLE 1. Total Least Squares Estimated Transformation Parameters

Parameter	Unit	Value	SD
T _x	Metre	-153.0127	11.6995
T _y	Metre	63.7693	22.4533
T _z	Metre	368.0575	21.1464
R _x	Radian	-1.07×10 ⁻⁰⁶	1.86×10 ⁻⁰⁶
R _y	Radian	-6.39×10 ⁻⁰⁶	3.32×10 ⁻⁰⁶
R _z	Radian	4.84×10 ⁻⁰⁶	3.50×10 ⁻⁰⁶
S	Part per million	-7.49×10 ⁻⁰⁶	1.82×10 ⁻⁰⁶

NB: T_x, T_y, T_z are the translation parameters. R_x, R_y and R_z are the rotational parameters and S is the scale factor

Analysis of Table 2 shows the coordinate differences (ΔE , ΔN) between the known and computed projected grid coordinates from TLS, RBFNN and TLS-RBFNN, respectively.

TABLE 2. Deviation of transformed test coordinates from existing coordinates

Test Point	TLS			RBFNN			TLS-RBFNN		
	$\Delta E(m)$	$\Delta N(m)$	HE (m)	$\Delta E(m)$	$\Delta N(m)$	HE (m)	$\Delta E(m)$	$\Delta N(m)$	HE (m)
T1	0.0793	1.7516	1.7534	0.2608	1.7445	1.7638	-0.2013	0.0387	0.2050
T2	0.3799	-0.3783	0.5361	0.5529	-0.0397	0.5543	-0.1618	0.4065	0.4376
T3	-0.7647	0.0937	0.7704	-0.6216	-0.4831	0.7873	0.2997	0.5741	0.6476
T4	-1.5285	-0.1834	1.5394	-0.8093	-0.3765	0.8926	0.0483	-0.3246	0.3282
T5	-1.2642	-1.4062	1.8909	-0.7940	-0.5080	0.9426	0.0580	1.1290	1.1305
Mean	-0.6196	-0.0245	1.2980	-0.2822	0.0674	0.9881	0.0086	0.3647	0.5498
SD	0.8292	1.1432	0.6074	0.6417	0.9559	0.4587	0.2012	0.5503	0.3631

NB: T1 to T5 are the point's identification for each test control points used

A careful study of the results in Table 2 reveals how much the computed Easting and Northing coordinates produced by TLS, RBFNN and TLS-RBFNN of the test points vary in conjunction with the ideal residual threshold value of zero. These values (ΔE , ΔN) give a better indication on the amount of discrepancies in the TLS, RBFNN and TLS-RBFNN computed coordinates compared to the known coordinates by way of errors. From Table 2, there is quantitative evidence of improvement of the TLS and RBFNN results by TLS-RBFNN. This could mean that the calibration capability of TLS-RBFNN architecture was better for the given training data and that has greater learning abilities compared to independently applying TLS and RBFNN to the data. In continuance of this, the result in Table 2 shows that the hybrid TLS-RBFNN model could generalise better with the test data than TLS and RBFNN. The SD values (Table 2) of the

coordinate differences estimated signify a practical expression for the precision of the computed coordinates given by TLS, RBFNN and TLS-RBFNN, respectively. In Table 2, it can be seen that TLS-RBFNN had the least SD values indicating the limit of the error bound by which every value within the computed test coordinates values varies from the most probable value.

Table 3 provides summary performance statistics of the total horizontal errors for TLS, RBFNN and TLS-RBFNN, respectively. From Table 3, it can be noticed that the hybrid TLS-RBFNN showed better results as compared to the other two techniques. Thus, TLS-RBFNN showed an improved percentage values of 76.870 and 43.140 in the transformation accuracy (RMSHE) for TLS and RBFNN. In terms of MHE, TLS-RBFNN improved TLS and RBFNN results by 74.820 and 43.830% respectively. For the maximum horizontal error, 76.040 and 63.330% were the percentage improvements by TLS-RBFNN. In the case of the minimum horizontal error, 33.110 and 34.930% were the improved percentage values. The inference made here is that TLS and RBFNN could not model the uncertainties of coordinates related to War Office 1926 and WGS84 in an effective way as compared to the TLS-RBFNN. On the basis of the SD values, TLS-RBFNN improved TLS and RBFNN by 24.430 and 9.560% (Table 3) with regards to the transformation precision.

TABLE 3. Total Horizontal Residuals of the Coordinate Differences Using the Three Methods

Performance Indicators	TLS (m)	RBFNN (m)	TLS-RBFNN (m)	Improvement for TLS (%)	Improvement for RBFNN (%)
RMSHE	1.4072	1.0699	0.6385	76.870	43.140
MHE	1.2980	0.9881	0.5498	74.820	43.830
Max HE	1.8909	1.7638	1.1305	76.040	63.330
Min HE	0.5361	0.5543	0.2050	33.110	34.930
SD	0.6074	0.4587	0.3631	24.430	9.560

NB: RMSHE, MHE, Max HE, Min HE and SD are defined in Equations (22) to (27)

Model Selection Criterion

To select the best performing model, the Bayesian Information Criterion (BIC) was explored. In this study, the HE values for each of the five test points were used to calculate the BIC value. The BIC was chosen because it tends to favour models with fewer parameters compared to other information criteria because its penalty term is smaller (Burnham and Anderson, 2002). The BIC is represented mathematically (Equation (28)) as

$$BIC = n \times \ln\left(\frac{SSE}{n}\right) + K \times \ln(n) \quad (28)$$

where n denotes the number of observations, SSE is the sum of squares of the residuals and K is defined as the penalty term which corresponds to the number of unknown parameters in a coordinate transformation model (Table 4). Thus, the SSE is acting as an optimality criterion to aid in the BIC model selection. Therefore, a model is selected as the most suitable candidate model if it gives the least estimated BIC value. With this in mind, it can be inferred from Table 4 that TLS-RBFNN is more feasible for transforming coordinates between WGS84 datum and War Office 1926 datum.

TABLE 4. **Bayesian Information Criterion Model Selection Results**

Method	K parameter	BIC Value
TLS	7	14.6817
RBFNN	2	3.8944
TLS-RBFNN	2	-1.2669

Coordinate Transformation Using Entire Dataset

Ghana GNSS reference station network at the moment covers only the Mid-Southern parts with expansion to cover the North and the whole country is yet to be done. Therefore, due to limited data availability in the study area (Ghana), a particular cross-validation technique was adopted to test the potential of the already determined optimal models of TLS, RBFNN and TLS-RBFNN using the entire dataset. In this approach, the testing data of five points were swapped in the optimized models with the 19 co-located points. Thus, the 19 points were applied as the testing data in the optimum RBFNN and TLS-RBFNN structure of [2-14-2] which has already been formed using the training data of 14 co-located points. Similarly, the transformation parameters determined using the TLS approach were also used to transform the whole dataset.

Mathematically, the test results produced using the entire dataset will give a better indication of all possible variations in the data and that will enable the model developer and users to ascertain the strength and generalisation capability of the developed models should the data size increase in the study area. In addition, it will provide a realistic estimate of the predictive potential of the TLS, RBFNN and TLS-RBFNN across the entire study area. This procedure has been adopted in several studies in order to check the overall generalisation of ANNs (Konaté, Pan, Khan, & Ziggah, 2015; Ziggah *et al.*, 2016b). The entire data testing results achieved by TLS, RBFNN and TLS-RBFNN are presented in Table 5.

Judging from the outcomes in Table 5, it can be inferred on the basis of the HE values that TLS-RBFNN model achieved satisfactory testing results than RBFNN and TLS. This demonstrates that TLS-RBFNN has good generalisation capabilities across the entire study area. Hence, it can evidently be stated that the TLS-RBFNN computed projected grid coordinates (testing outputs) are in better agreement to the known projected grid coordinates.

In addition, the lowest SD values (Table 5) obtained by TLS-RBFNN revealed its generalisation superiority to the other methods. These SD values show the range of precision of the TLS, RBFNN and TLS-RBFNN computed projected grid coordinates on a normal distribution curve. In line with this, it can be stated that the TLS-RBFNN transformation results are more precise and accurate than the TLS and RBFNN. Intuitive interpretation of Figure 4 shows that the TLS-RBFNN generalised better across the entire data for the study area than the TLS and RBFNN, respectively.

TABLE 5. Deviation of Transformed Test Coordinates Using the Whole Data set

Point	TLS			RBFNN			TLS-RBFNN		
	ΔE (m)	ΔN (m)	HE(m)	ΔE (m)	ΔN (m)	HE(m)	ΔE (m)	ΔN (m)	HE(m)
T1	-0.3692	1.1937	1.2495	-0.4021	0.8442	0.9351	-0.1086	-0.4933	0.5051
T2	-0.1522	0.9330	0.9454	-0.2589	0.8414	0.8803	-0.0110	0.3605	0.3607
T3	0.4910	-1.6776	1.7479	0.2246	-1.1899	1.2109	-0.0064	-0.0365	0.0371
T4	-0.6727	-0.6522	0.9370	-0.5209	-1.3907	1.4851	-0.0978	-0.2829	0.2993
T5	0.0600	-0.9153	0.9173	-0.2790	-0.4701	0.5466	-0.0894	-0.6217	0.6281
T6	0.2516	0.5738	0.6265	0.0827	0.8848	0.8886	0.2740	0.9442	0.9831
T7	0.3772	-0.1226	0.3966	0.2504	-0.1123	0.2744	0.0615	0.5202	0.5238
T8	0.3940	-0.3777	0.5458	0.1609	-0.1136	0.1969	0.3702	0.1237	0.3903
T9	-0.4205	-0.4637	0.6260	-0.7729	-0.1023	0.7796	-0.6297	-0.4714	0.7866
T10	0.3165	0.7873	0.8485	0.3901	0.0289	0.3912	-0.0109	0.3524	0.3526
T11	-0.3867	0.5759	0.6937	-0.5904	0.1356	0.6057	0.1550	0.2527	0.2964
T12	0.6269	0.0195	0.6272	0.3452	-0.1077	0.3616	0.3119	-0.5031	0.5919
T13	-0.4063	0.3972	0.5682	-0.6027	0.3222	0.6834	-0.2460	-0.1002	0.2656
T14	-0.1628	-0.2977	0.3393	-0.4740	0.1553	0.4988	0.0473	-0.0497	0.0686
T15	0.0793	1.7516	1.7534	0.2608	1.7445	1.7638	-0.1920	-1.5900	1.6016
T16	0.3799	-0.3783	0.5361	0.5529	-0.0397	0.5543	-0.1171	0.7646	0.7735
T17	-0.7647	0.0937	0.7704	-0.6216	-0.4831	0.7873	0.3221	0.5301	0.6203
T18	-1.5285	-0.1834	1.5394	-0.8093	-0.3765	0.8926	-0.0032	0.0248	0.0251
T19	-1.2642	-1.4062	1.8909	-0.7940	-0.5080	0.9426	0.0570	1.1290	1.1304
SD	0.5952	0.8661	0.4816	0.4589	0.7321	0.4001	0.2337	0.6379	0.3964

NB: T1 to T19 are the point's identification defining each test control points used

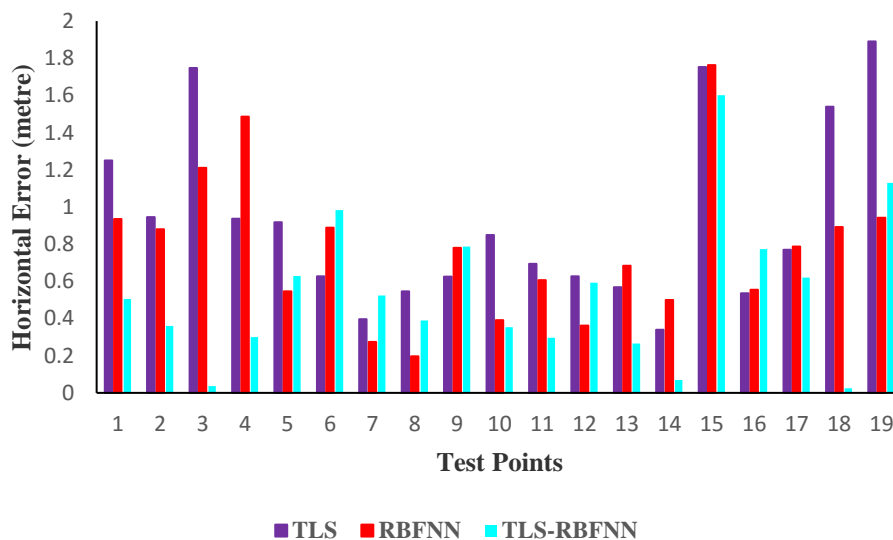


FIGURE 4. Horizontal Displacement of the Entire Data (testing data)

As evident from Table 6, the TLS-RBFNN had a transformation accuracy of 0.6628 m while 1.0362 m and 0.8652 m were respectively produced by TLS and RBFNN. In percentage wise, the

TLS and RBFNN transformation accuracies were improved by 37.34 and 20.24, respectively. These signify the degree at which the hybrid TLS-RBFNN model could improve the transformation results in the study area rather than independently applying TLS and RBFNN. Taking into account the average dispersion of horizontal errors (MHE), it can be seen that when TLS-RBFNN was applied the TLS and RBFNN results were improved by 38.52% and 23.36%. The maximum and minimum HE (Table 6), on the other hand, signify the quality of the results produced by the three methods with respect to the range of error achievable when TLS, RBFNN and TLS-RBFNN were applied in the study area. An observation from Table 6 shows that independently applying TLS and RBFNN in the study area will achieve maximum HE values of 1.8909 m and 1.7638 m with 0.3393 m and 0.1969 m being the minimum. However, should the TLS-RBFNN be applied, TLS and RBFNN results could be improved by 28.93% and 16.22% for their maximum HE. In relation to minimum HE, 31.42% and 17.18% could be obtained. On account of the SD calculated values (Table 6), it could be seen that a transformation precision of 0.3964 m was achieved by TLS-RBFNN while 0.4816 m and 0.4001 m were realised by TLS and RBFNN respectively. The TLS-RBFNN results indicate percentage improvement of TLS and RBFNN results by 8.52% and 0.37%.

TABLE 6. Total Horizontal Residuals using the Entire data

Performance Indicator	TLS (m)	RBFNN(m)	TLS-RBFNN(m)	Improvement for TLS (%)	Improvement for RBFNN (%)
RMSHE	1.0362	0.8652	0.6628	37.340	20.240
MHE	0.9242	0.7726	0.539	38.520	23.360
Max HE	1.8909	1.7638	1.6016	28.930	16.220
Min HE	0.3393	0.1969	0.0251	31.420	17.180
SD	0.4816	0.4001	0.3964	8.520	0.370

NB: RMSHE, MHE, Max HE, Min HE and SD are defined in Equations (22) to (27)

The computed BIC values (Table 7) based on the HE (Table 5) of the entire data set showed that the combination of TLS and RBFNN had a better capability of producing improved transformation results than when they are separately applied. This is because among the three methods TLS-RBFNN had the least BIC value and thus was selected as the better technique over TLS and RBFNN, respectively. Hence, on the basis of the quantitative analyses presented in this study, it can logically be stated that the potential of the hybrid TLS-RBFNN approach in the coordinate transformation process for Ghana geodetic reference network has been duly investigated.

TABLE 7. Bayesian Information Criterion Model Selection Using Whole Data

Method	K Parameter	BIC Value
TLS	7	12.6190
RBFNN	2	-2.2856
TLS-RBFNN	2	-12.4093

CONCLUSIONS AND RECOMMENDATION

This study has demonstrated the potential and resulting consequences of using the TLS integrated with RBFNN than applying the TLS and RBFNN independently in coordinate transformation

process. The proposed TLS-RBFNN has the ability to modify its behaviour to improve its learning ability and to generalise well to unseen data (test data). From the results, the obtained transformation accuracy (RMSHE) values for TLS and RBFNN were improved by TLS-RBFNN by 37.3% and 20.2%. Similar performance enhancement were observed for the transformation precision (SD values), maximum and minimum recorded horizontal errors as well as the average horizontal error (MHE). Future research works will focus on generalising the proposed hybrid approach across the entire country (Ghana) with more data availability upon completion of the GNSS reference stations in the near future. Additionally, it is also recommended that other artificial intelligence methods be applied with the TLS to help select the best performing hybrid transformation model for Ghana.

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